

Unit 11

Solving Linear Inequalities and Interval Notation

Math Essentials

1

Mathematically, how do we represent the statement "Jim ate no more than 2 cookies"?

$<$ "is less than" \leq "is less than or equal to"
 $>$ "is greater than" \geq "is greater than or equal to"

How do we draw a picture to represent this statement?

FIGURE 9 The interval $(-\infty, 2]$

2

Translate to inequalities.

- Jim's age exceeds 21 yrs. $j > 21$
- Jim is at least 21 yrs old. $j \geq 21$
- Jim is no less than 21 yrs old. $j \geq 21$
- Jim's age is at most 21 yrs. $j \leq 21$
- Jim is no more than 21 yrs old. $j \leq 21$

3

Practice 11.1

Write an inequality for the following statement.

y is greater than 6

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4

Practice 11.2

Rewrite the set by listing its elements. Make sure to use the appropriate set notation.

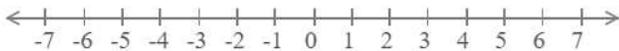
$s = \{x \mid x \text{ is an integer and } -3 \leq x < 2\}$

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5

Practice 11.3

Graph the set $\{x \mid x < -4\}$ on the number line. Then, write the set using interval notation.

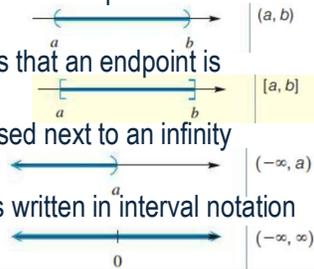


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6

Interval notation uses symbols like $()$, $[]$

- A parenthesis indicates that an endpoint is not included.
- A square bracket indicates that an endpoint is included.
- A parenthesis is always used next to an infinity symbol, $-\infty$ or ∞ .
- The set of real numbers is written in interval notation as $(-\infty, \infty)$.



7

Practice 11.4

Graph each of the following inequalities and write it using interval notation.

1. $a > 0$
2. $0 \leq y \leq 1$
3. $b \leq -5$ or $b > 5$

8

Practice 11.5

Solve $x - 2 < -6$
and graph the solution set.

9

Addition Principle for Inequalities

For any real numbers a , b , and c :

$$a < b \text{ is equivalent to } a + c < b + c$$

$$a > b \text{ is equivalent to } a + c > b + c$$

(similar for \leq and \geq)

10

Practice 11.6

Solve each inequality for the variable. Simplify your answer.

1. $12 \leq v + 7$

2. $4 + y \geq 3$

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11

Multiplication Principle for Inequalities

•What about $4x < 8$?

For any real numbers a , b , and POSITIVE numbers c :

$$a < b \text{ is equivalent to } ac < bc$$

$$a > b \text{ is equivalent to } ac > bc$$

(similar for \leq and \geq)

12

Practice 11.7

Solve the inequality for x .

1. $5x + 1 > 66$

2. $-1 + 4x > 19$

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13

- Consider the true statements: $-2 < 3$
 - Multiply both sides by 4. Is it still true?
 - Multiply both sides by -4 . Is it still true?

14

For any real numbers a , b , and **NEGATIVE** numbers c :

- $a < b$ is equivalent to $ac > bc$
 - $a > b$ is equivalent to $ac < bc$
- (similar for \leq and \geq)

15

When multiplying (or dividing) by a negative number, **REVERSE** the inequality sign!!

16

Practice 11.8

Solve the inequality for the variable.

1. $-19 \leq -2x - 31$

2. $-\frac{7w}{3} - 6 \geq -12$

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17

Practice 11.9

Solve each inequality for v . Simplify your answer.

1. $3v - 4 \geq 6v - 19$

2. $-8v + 5(v - 2) > -34$

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18

Practice 11.10

Solve the inequality for u . Simplify your answer.

$$\frac{1}{8}(u - 2) > 5u - \frac{9}{4}$$

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19

Practice 11.11

Solve the inequality and graph the solution set.

1. $-3(x - 6) > 2x - 2$
2. $-(9 + x) - 5 + 4x \geq 4$
3. $5 - \frac{7}{2}y < \frac{3}{4}y$

20

Practice 11.12

Solve the inequality for x . Graph your solution on the number line.

$$2x + 1 \geq 9$$

or $3x - 2 \leq -14$

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21

Practice 11.13

Solve the inequality for v . Write the solution in interval notation. If no solution, write \emptyset

1. $2v - 4 \leq -16$ or $4v + 1 \geq 25$
2. $3x + 4 < -5$ and $4x - 3 \geq 17$

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22

Practice

Solve the inequality for x . Write the solution in interval notation.

$$-36 < 7x - 1 < 6$$

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23

Practice 11.14

Carmen got a prepaid debit card with \$15 on it. For her first purchase with the card, she bought some bulk ribbon at a craft store. The price of the ribbon was 16 cents per yard. If after that purchase there was \$13.08 left on the card, how many yards of ribbon did Carmen buy?

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24

Practice 11.15

To rent a certain meeting room, a college charges a reservation fee of \$36 and an additional fee of \$5.60 per hour. The chemistry club wants to spend at most \$75.20 on renting the meeting room. What are the possible amounts of time for which they could rent the meeting room? Use h for the number of hours the meeting room is rented, and solve your inequality for h .

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25

Practice 11.16

Keith is choosing between two exercise routines. In Routine #1, he burns 48 calories walking. He then runs at a rate that burns 12.9 calories per minute. In Routine #2, he burns 22 calories walking. He then runs at a rate that burns 15.5 calories per minute. For what amounts of time spent running will Routine #1 burn at most as many calories as Routine #2? Use t for the number of minutes spent running, and solve your inequality for t .

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26

Steps for solving word problems with inequalities.

1. **Read** the problem carefully. What is given? What do we need to find?
2. **Assign a variable** to represent the unknown value. Make a sketch, diagram, or table, as needed. Identify any other unknowns in terms of the defined variable.
3. **Write an inequality** using the variable.
4. **Solve the inequality.**
5. **State the answer.** Label it appropriately. Does it seem reasonable?
6. **Check** the answer in the words of the original problem.

27

Practice 11.17

A rental company charges \$20 to rent a chain saw, plus \$9 per hr. Tom can spend no more than \$65 to clear some logs from his yard. What is the maximum amount of time he can use the rented saw?

28

Practice 11.18

Newlyweds Bryce and Lauren need to move their belongings to their new apartment. They can rent a truck from U-Haul for \$29.95 per day plus 28 cents per mile or from Budget Truck Rentals for \$34.95 per day plus 25 cents per mile. After how many miles (to the nearest mile) would the Budget rental be a better deal than the U-Haul rental?

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29
